An exact approach for a bicriteria maximal SRLG-disjoint / minimal cost path pair problem in telecommunication networks.

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Abstract: Routing mechanisms in transport telecommunication networks, namely optical networks, require that very high levels of network service availability be maintained in the event of failures. This results from the requirements of the of Services Levels Agreements, to be provided to the customers by the network operators and from the enormous amounts of traffic that can be lost in the event of failures in the physical network, such as fiber cuts, switch or software failures. One of the most commonly used fault recovery mechanisms is global path protection, in which a pair of paths (corresponding to end-to-end routes), the active path (AP), that carries traffic flow under normal operating conditions and the backup path (BP), which is the path that carries that traffic when some failure affects the AP, are computed and established simultaneously, so that the availability of the services supported by the pair be maximized in the event of certain failures. In the design of routing mechanisms with built-in survivability objectives, and taking into account the multi-layered structure of telecommunication networks, the concept of shared risk link group (SRLG) is used, which may be defined as a group of logical links that share a common risk of failure. Usually, the network designer, based on the information about the SRLGs, seeks to calculate a pair of paths which are SRLG-disjoint, ensuring that no single fault of the AP will affect the BP, a NP-complete problem as shown in [1]. However, there may arise situations in which no SRLG-disjoint path pair can be calculated, a case in which the aim of the routing procedure may consist of finding a maximally SRLG-disjoint path pair that is a path pair with the minimum number of common SRLGs. Moreover, a key concern, in a routing model, is bandwidth usage optimization, seeking to optimize the use of bandwidth resources throughout the network links, in order to achieve the maximal possible network traffic carrying capability. This is usually represented in terms of additive path cost functions, such that the cost of using a link is some function of its capacity and used bandwidth. These objectives make that a typical formulation of the routing problem with path protection is the lexicographic calculation of a pair of paths which are maximally SRLG-disjoint and, as secondary objective, have minimal total cost. Several heuristic algorithms for finding totally SRLG-disjoint path pairs have been proposed, the performance of which, in terms of accuracy, is usually evaluated by comparison with exact solutions from Integer Linear Programming formulations, for problems tested in reference networks. Also a few heuristics were proposed for tackling maximally SRLG-disjoint path pairs of minimal cost lexicographic problems, considering variants of the objective functions or of the constraints and various resolution approaches. In particular, [2] presents two heuristics for tackling a lexicographic formulation of this type of problem which includes as additional objectives, of highest priority, that the paths are maximally node and arc disjoint.

In this work we address a bi-criteria formulation of the maximal SRLG-disjoint / minimal cost path pair problem, in the context of resilient routing design with path protection and propose an approach for exact calculation of non-dominated solutions to this problem. The resolution method, for the formulated bi-criteria optimization problem, is based on an exact combinatorial algorithm already developed, that enables the optimal solution to the lexicographic version of the problem, to be to obtained. This base algorithm is a lexicographic minimal label-minimal cost path pair algorithm which combines a path ranking method - where possible paths are ranked by increasing order of cost by using the ranking algorithm [3] - and a path labeling method. Note that the lexicographic formulation is the most commonly used by network designers. However, we think that by considering the proposed bi-criteria formulation, the choices of the network designer are clearly widened, enabling the exploration of trade-offs between the minimisation of failure risks and path pair load costs, which may be conflicting objectives. Therefore, we extended our lexicographic algorithm, in order to obtain exact non-dominated solutions, of the bi-criteria problem, in a set of solutions generated by the lexicographic algorithm. Note that these nondominated solutions constitute a sub-set of the whole non-dominated solution set. We will consider two variants of the selection process of non-dominated solutions, to be presented to the network designer, so that he/she may make a final choice according to his/her system of preferences. In the first variant, the non-dominated solutions are selected in the set of all solutions generated throughout the algorithm execution, until the lexicographic optimum was found. In the second variant we will seek non-dominated solutions which satisfy a pre-defined upper-bound in terms of risks common to the AP and the BP, that is, by considering a small relaxation with respect to the minimal number of common risks. The selection of the sub-set of non-dominated solutions, in both variants, is performed by using the algorithm in (4).
We will begin by reviewing the base lexicographic algorithm –a lexicographic minimal label-minimal cost path pair algorithm – which combines a path ranking method and a path labeling method. Some experiments for evaluating the algorithm performance, obtained in virtual networks constructed over the classical US–NSF reference network topology, considering various distributions of random SRLG assignments to the links and given the link occupancies, also randomly generated, will also be shown. Also preliminary results concerning the performance, in terms of computing times, and typical trade-offs obtained with the bi-criteria solutions, will be presented and discussed, for some reference test networks. Finally, some conclusions from this on-going research theme, as well as further work, will be outlined.

Palabras Clave: Resilient routing models; Bi-criteria optimization; Telecommunication network design; Pathpair ranking.

Referencias:

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Introduction and Definitions

- Routing mechanisms in transport telecommunication networks, require that very high levels of network service availability be maintained in the event of failures
- A most common resilient routing mechanisms is global path protection, in which a pair of paths, the *active path* (AP) and the *backup path* (BP), ie the path which carries the traffic when some failure affects the AP, are established simultaneously
- In this context the concept of SRLG (Shared Risk Link Group) is used: the set of logical links (arcs) which share a common risk of failure.

Definitions

- G=(N,A): **directed network**, A$\subseteq$N×N (arc set), |N|=n (node set), |A|=m;

  s $\in$ N (t $\in$ N): source (terminal) node.
- P: set of **feasible paths** in G from s to t, for the required service.
- Let L be the **set of network risks**
- $A_l \subseteq$ A the set of all arcs which are affected by risk l – this set defines the **$SRLG_l$**

  and (i, j) be an arc:

  $c_{ij} \in R_0^+$: **cost** of using(i,j)

  $L_{ij} = \{l_{ij}^1, l_{ij}^2, \dots, l_{ij}^k\} \subseteq$ L: **set of all risks which may affect** (i,j).
- Given a path p $\in$ P, we denote:

- its **cost**, by c(p) = $\sum_{(i,j)\epsilon\, p} c_{ij}$

- **the set of risks which affect p**, by l(p) = $\bigcup_{(i,j)\epsilon\, p} L_{ij}$

Definitions

- For pairs of paths in P, (p,q):

- **cost** c(p,q)=c(p)+c(q)

- the number of **risks common to both paths** l(p, q) = |l(p) ∩ l(q)|

➢ The most usual formulation of resilient routing, in network design is the *SRLG (or risk)-disjoint min cost* path pair problem:

min c(p,q)

s.t. l(p,q)=0 p, q ∈ P

▪ However, in some networks this problem may be infeasible!

Introduction and Definitions

- Therefore a general formulation, is the *lexicographic maximal SRLG (or risk)-disjoint min cost path pair* problem - **LMRDPMCP** (it is NP complete cf Hu [3]):

$$\text{lex min}_{p,q \text{ e } P}(l(p,q), c(p,q))$$

that is,

$$\min c(p,q)$$

$$\text{s.t. } l(p,q)=l^* \text{ with } l^* = \min l(p,q) \quad p,q \in P$$

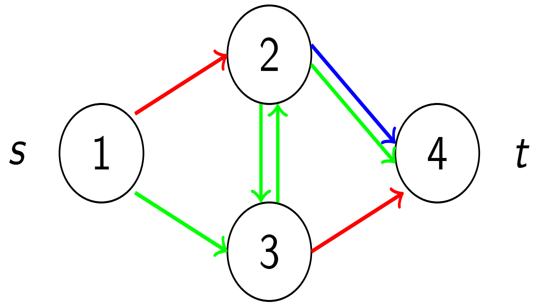
- An exact algorithm for solving this problem, described next, which combines a path ranking method and a path labeling method, was developed.

Introduction and Definitions

LMRDPMCP – Example

Let

- $c_{ij} = 1$
- L_{ij} be defined by the different colors.



The optimal pair of paths is $(\langle 1, 3, 2, 4\rangle, \langle 1, 3, 4\rangle)$, with cost 5 and 1 shared risk.

Bicriteria maximal SRLG-disjoint / minimal cost path pair

- However, we think that by considering a ***bi-criteria formulation***, the choices of the network designer are clearly widened, *enabling the exploration of trade-offs between the minimisation of failure risks and path pair load costs, which may be conflicting objectives:*

$$\min Z_1 = c(p,q)$$

$$\min Z_2 = l(p,q)$$

s.t. p, q e P.

- The resolution method, for this bi-criteria optimization problem, is based on the lexicographic combinatorial algorithm, LMRDPMCP.

Bicriteria maximal SRLG-disjoint / minimal cost path pair

LMRDPMCP algorithm

1.An upper bound on the number of shared risks, RiskUB, is set.

2.Possible paths p_1 are listed by increasing order of cost (ranking algorithm [4]).

3.For each path p_1, the path q such that

lex min $(z_1(p_1, q), z_2(p_1, q))$ s.t. $q \in P$

is computed according to the *labelling algorithm LLC*.

This step is repeated for the next path in terms of cost ranking until the optimum is found.

Note: RiskUB is defined as the number of risks of the shortest path from s to t.

Bicriteria maximal SRLG-disjoint / minimal cost path pair

Labelling algorithm LLC

Node labels: Node i is associated with label $L_i =$

$(\pi_i^c,$: *cost of the path from s to i*
$\pi_i^r,$: *set of risks in* p_1 *that appear also in the path q from s to i*
$\pi_i^l)$: *# of risks in* p_1 *that appear also in the path q from s to i*

Algorithm:

if $(i,j) \in A$ **then**
  $\delta_j^c \leftarrow \pi_i^c + c_{ij}$; $\delta_j^r \leftarrow \pi_i^r \cup L_{ij} \cap p_1$; $\delta_j^l \leftarrow |\delta_j^r|$
  if $\delta_j^l \leq RiskUB$ **then**
    if $(\delta_j^c, \delta_j^r, \delta_j^l)$ *is not dominated* **then**
      $L_j \leftarrow (\delta_j^c, \delta_j^r, \delta_j^l)$
      if $j = t$ **then** Update $RiskUB$ and $(Bestp, Bestq)$;

Dominance between labels: L_x dominates L'_x iff $\pi_x^c \leq \pi_x'^c$, $\pi_x^r \subseteq \pi_x'^r$, $\pi_x^l \leq \pi_x'^l$ and $L_x \neq L'_x$

Bicriteria maximal SRLG-disjoint / minimal cost path pair

- To obtain exact non-dominated solutions of the bi-criteria problem, from the set of solutions generated by the lexicographic algorithm, we consider *two variants* of the selection process of non-dominated solutions:
  - non-dominated solutions are *selected in the set of all solutions generated* throughout the algorithm execution, until the lexicographic optimum was found;
  - we will seek only non-dominated solutions which satisfy a pre-defined *upper-bound in terms of risks common* to the AP and the BP.
- The *selection* of the sub-set of non-dominated solutions, in both variants, is performed by using the algorithm in [6]

- These n.d. solutions *are presented to the network designer*, so that he/she may make a final choice according to his/her system of preferences.

Computational experiments

- Reference test networks were considered:

Network	n	m	$\delta = m/n$	$\|L\|$	$\alpha = \|L\|/m$
NSFFixedLabels [1]	11	26	0.8	21	1.5
NSFRandomLabels [1]	14	21	1.3	15, 20, 25	1, 4
NobelEu [5]	28	41	2.9	15, 20, 25	1, 4
Cost266 [5]	37	57	3.1	15, 20, 25	1, 4

10 sets of parameters, and $m/2$ $s-t$ pairs.

Computational experiments

- Costs c_{ij} represent link occupancies and are given by $c_{ij} = 1/b_{ij}$, where the available bandwidths b_{ij} are *randomly generated* according to the distributions:

	I_0	I_1	I_2	I_3
D1	25%	25%	25%	25%
D2	70%	15%	10%	5%
D3	18%	18%	18%	46%

in the intervals:

$$\begin{aligned} I_i &= \{2+2k : k = 20i, \ldots, 20(i+1)-1\} \quad (i = 0, 1, 2) \\ I_3 &= \{2+2k : k = 60, \ldots, 78\} \end{aligned}$$

- The SRLGs L_{ij} are *uniformly generated* between 1 and |L| = 15, 20, 25, with mean number of SRLGs per link α = 1, 4

Computational experiments (NSF network)

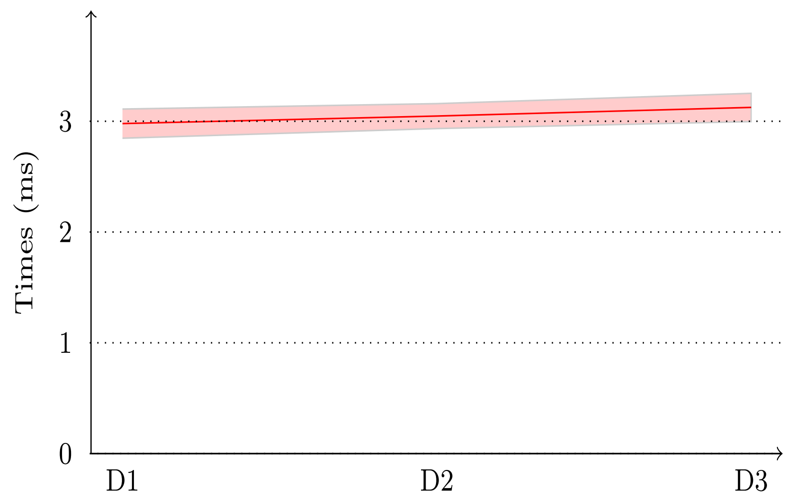


Figure 1: Average CPU times and 95% confidence interval in NSF network with fixed SRLGs

Computational experiments (NSF network)

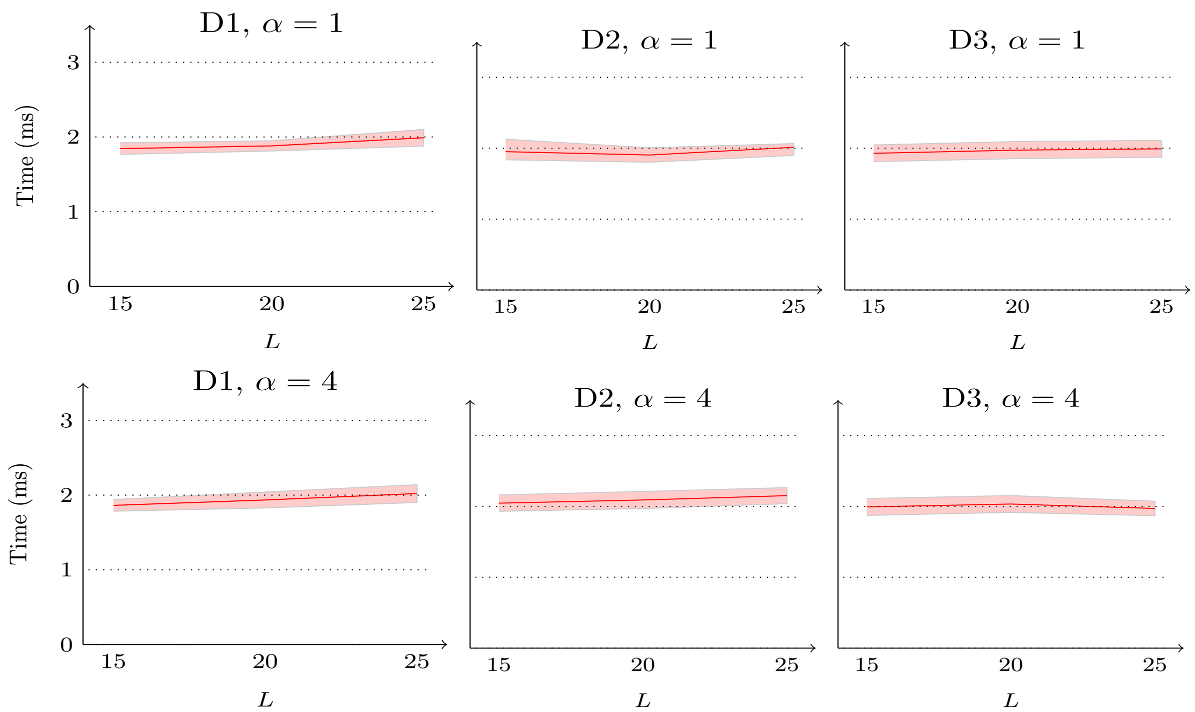


Figure 2: Average CPU times and 95% confidence interval in NSF network with random SRLGs

Computational experiments (NobelEU network)

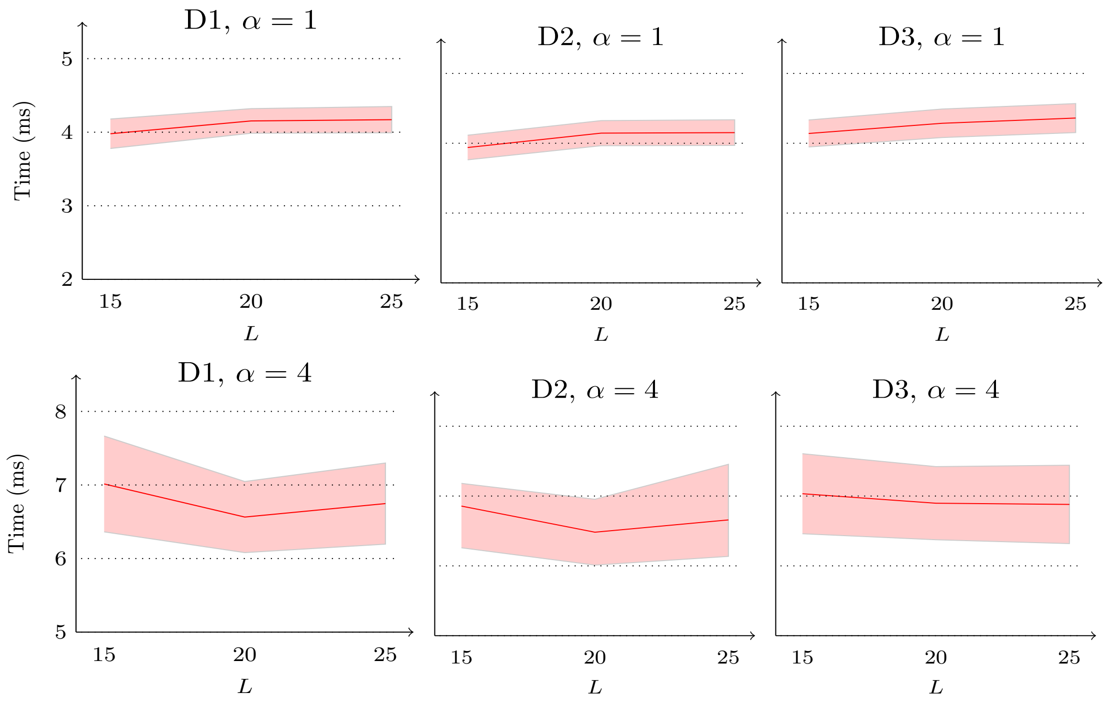


Figure 3: Average CPU times and 95% confidence interval in NobelEU network with random SRLGs

Computational experiments (Cost266 network)

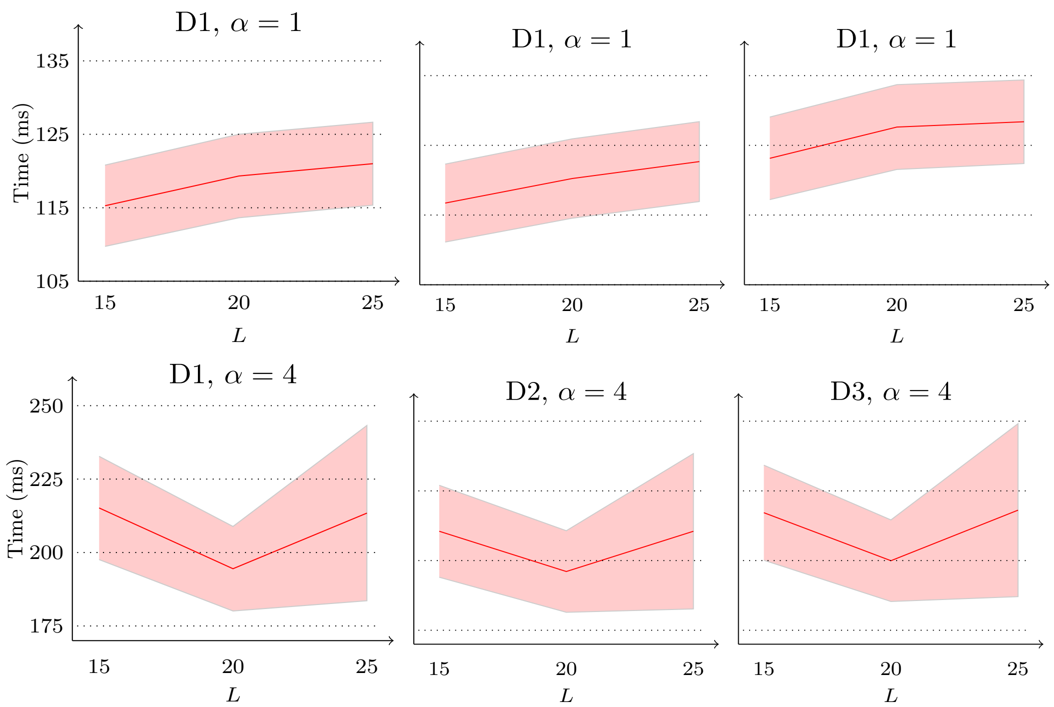


Figure 4: Average CPU times and 95% confidence interval in Cost266 network with random SRLGs

Computational experiments (selected instance for NSF network)

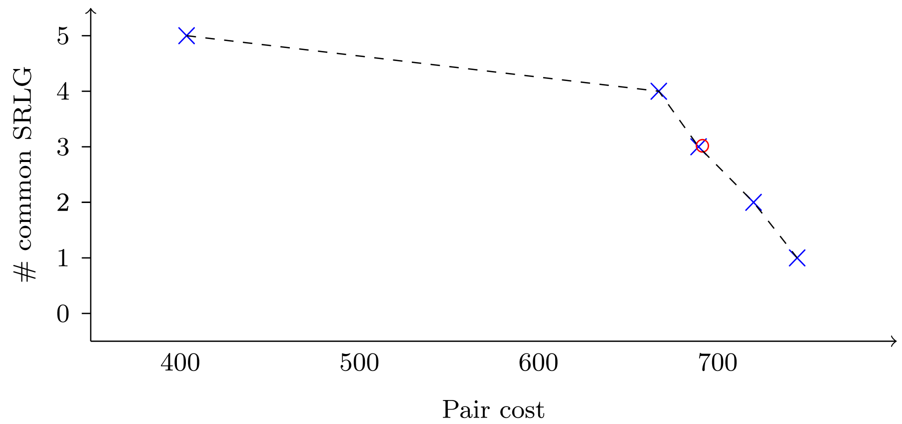


Figure 5: Solutions for selected problems with NSF network and random SRLGs

Pair cost	# common SRLG	
744.409	1	(a)
720.061	2	(a)
689.379	3	
667.160	4	
403.532	5	

Table 1: Objective values for the selected problem in NSF network with random SRLG in Figure 5
(a) N.d. solutions presented to the network designer, in the 2^{nd} variant ($\Delta l^* \leq 2$)

Computational experiments (selected instance for Cost266 network)

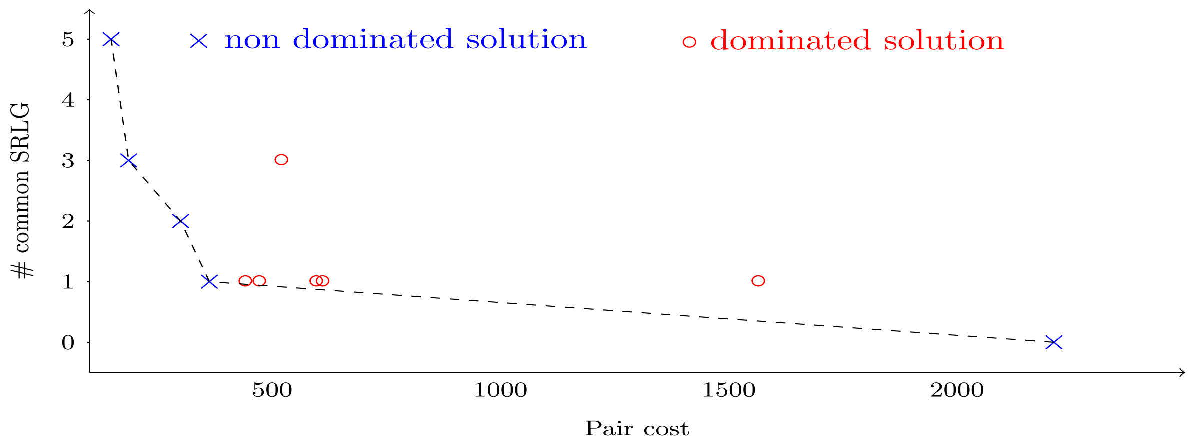


Figure 6: Solutions for selected problems with network Cost266 and random SRLGs

Pair cost	# common SRLG	
2212.531	0	(a)
362.672	1	(a)
299.596	2	
185.811	3	
147.333	5	

Table 2: Objective values for the selected problem in Cost266 network with random SRLG in Figure 6

(a) N.d. solutions presented to the network designer, in the 2^{nd} variant ($\Delta l^* \leq 2$)

Conclusions

- We presented a *bi-criteria formulation* for the *maximal SRLG-disjoint / minimal cost path pair problem* in telecommunication networks
- We showed how exact non-dominated solutions of the bi-criteria problem, can be obtained from the set of solutions generated by the proposed lexicographic algorithm LMRDPMCP, by considering *two variants* of the selection process
- These n.d. solutions *are presented to the network designer*, thus *enabling the exploration of trade-offs between the minimisation of failure risks and path pair load costs*

- The *computational efficiency* of the base algorithm and of the solution selection procedure make this approach suitable for application in most reference test networks scenarios.

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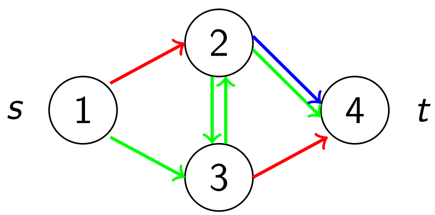
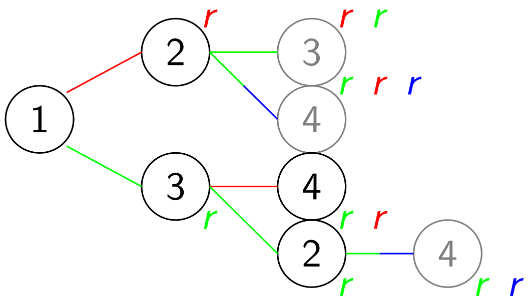
APPENDIX

Working of the LMRDPMCP algorithm

Example



Given the (shortest) path $\langle 1, 2, 4 \rangle$ and $RiskUB = 3$.



The new best pair of paths is $(\langle 1, 2, 4 \rangle, \langle 1, 3, 4 \rangle)$, with cost 4 and 2 shared risks.

APPENDIX

ILP formulation of the LMRDPMCP problem

Decision variables

- $x_{ij}^k = \begin{cases} 1 \text{ if arc } (i,j) \text{ is in the path } p_k \\ 0 \text{ otherwise} \end{cases}, \quad \forall (i,j) \in A, \quad k = 1,2$

- $v_l^k = \begin{cases} 1 \text{ if } l \text{ is a risk in path } p_k \\ 0 \text{ otherwise} \end{cases}, \quad \forall l \in L, \quad k = 1,2$

- $v_l = \begin{cases} 1 \text{ if risk } l \text{ is shared by the pair} \\ 0 \text{ otherwise} \end{cases}, \quad \forall l \in L$

Objective functions

- $z_1 \equiv \sum_{l \in L} v_l$

- $z_2 \equiv \sum_{(i,j) \in A} c_{ij}(x_{ij}^1 + x_{ij}^2)$

APPENDIX

ILP formulation of the LMRDPMCP problem

1. $\sum_{(i,j)\in A} x_{ij}^k - \sum_{(j,i)\in A} x_{ji}^k = b_i, \; k = 1, 2$, with
$$b_i = \begin{cases} 1, & i = s \\ 0, & i \in \mathcal{N} - \{s, t\} \\ -1, & i = t \end{cases}$$
Flow conservation for all nodes and paths start at s and end at t.
2. $\sum\limits_{(i,j)\in A_l} x_{ij}^k \leq \min\{n-1, |A_l|\} v_l^k, \quad l \in L, \quad k = 1, 2$
Assure that for each risk and each path, an arc with that risk is only in the solution if the associated risk variable is 1.
It also implies that the number of arcs in each path in the solution with each risk cannot exceed $n-1$, nor the number of arcs with that risk in the network.
3. $v_l^1 + v_l^2 - v_l \leq 1, \; l \in L$
Relate the risk variables.
4. $x_{ij}^k, v_l^k, v_l \in \{0, 1\}, \; (i,j) \in A, \; l \in L, \; k = 1, 2$

APPENDIX

ILP formulation of the LMRDPMCP problem

Inspired by [3], we can solve

$$
\begin{array}{ll}
\min & \sum_{(i,j)\in A} c_{ij}(x_{ij}^1 + x_{ij}^2) \\
\text{s.t.} & \sum_{(i,j)\in A} x_{ij}^k - \sum_{(j,i)\in A} x_{ji}^k = b_i \\
& \sum_{(i,j)\in A_l} x_{ij}^k \leq \min\{n-1, |A_l|\} v_l^k \\
& v_l^1 + v_l^2 - v_l \leq 1 \\
& \sum_{l\in L} v_l = l^* \\
& x_{ij}^k, v_l^k, v_l \in \{0,1\}
\end{array}
$$

where

$$
l^* = \begin{array}{ll}
\min & \sum_{l\in L} v_l \\
\text{s.t.} & \sum_{(i,j)\in A} x_{ij}^k - \sum_{(j,i)\in A} x_{ji}^k = b_i \\
& \sum_{(i,j)\in A_l} x_{ij}^k \leq \min\{n-1, |A_l|\} v_l^k \\
& v_l^1 + v_l^2 - v_l \leq 1 \\
& x_{ij}^k, v_l^k, v_l \in \{0,1\}
\end{array}
$$

APPENDIX

Features of the computational system

`NA` code in C language, running over openSUSE Leap 42.2.
`ILP` formulation solved with CPLEX 12.7.

Computer characteristics:

- Intel® i7-6700 Quad core, cache of 8Mb
- Processor 3.4 GHz
- 16 Gb of RAM

APPENDIX

Comparison of computational performance of LMRDPMCP with ILP formulation in CPLEX (cf comunication presented at INOC 2017, Lisbon, feb 2017)

- The algorithm LMRDPMCP is significantly more efficient than the ILP formulation for all instances in networks NSF, both for fixed and random labels, and in networks Nobel EU, for all considered distributions of label assignments.

- Only for the largest and more dense reference networks Cost266, LMRDPMCP is less efficient than the ILP formulation.

- As *future work* a new, more efficient, variant of LMRDPMCP, is being developed.