GRUPO ESPAÑOL DE DECISIÓN MULTICRITERIO

X Reunión del GEDM – Universidad CEU San Pablo, Madrid

VETO VALUES IN GROUP DECISION MAKING WITH INCOMPLETE INFORMATION WITHIN MAUT

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1. Introduction

Muti-attribute utility theory (MAUT)

Group decision-making context

Partial, imprecise or incomplete information

Vetoes

DECISION-MAKING PROBLEM

Methodology

- Extension of the additive model (MAUT) to deal with vetoes
- Dominance measuring methods → ranking of alternatives
- Ranking aggregation (order explicit algorithm) → group ranking

1. Introduction

ILLUSTRATIVE EXAMPLE

Selection of optimal remedial strategies for restoring radionuclide contaminated aquatic ecosystem and drainage areas

EUROPEAN PROJECTS

- MOIRA, 4th EU Framework Programme. 1996-1998.
- COMETES, International Cooperation Programme INCO-COPERNICUS. 1998-2001.
- EVANET-HYDRA, 5th EU Framework Programme. 2001-2004.
- EURANOS, 6th EU Framework Programme, 2004-2008.

1. Introduction

LAKE PALANCOSO (EXTREMADURA, SPAIN)

Surface area of roughly 100,000 m^2
Catchment area is 5 times the size of the lake
Depth is highly variable over the year
Situated at 270 m above sea level

Not a source of drinking water,
Status of **special protection area for birds**

Attracts a lot of **tourists**, due to birdwatching, some of the birds being in danger of extinction.

Hypothetical **severe accident** at the **Almaraz nuclear power plant** (Spain).

↓

Contamination with Cs^{137} between 104 and 105 Bq/m^2
Sr^{90} around 3.000 Bq/m^2.

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2. **Problem structuring**
3. **Countermeasures and their impacts**
4. **Quantifying DM's preferences** (component utilities, weights and vetoes)
5. **Evaluation of alternatives** (dominance measuring methods)
6. **Aggregating the rankings**

2. Problem structuring

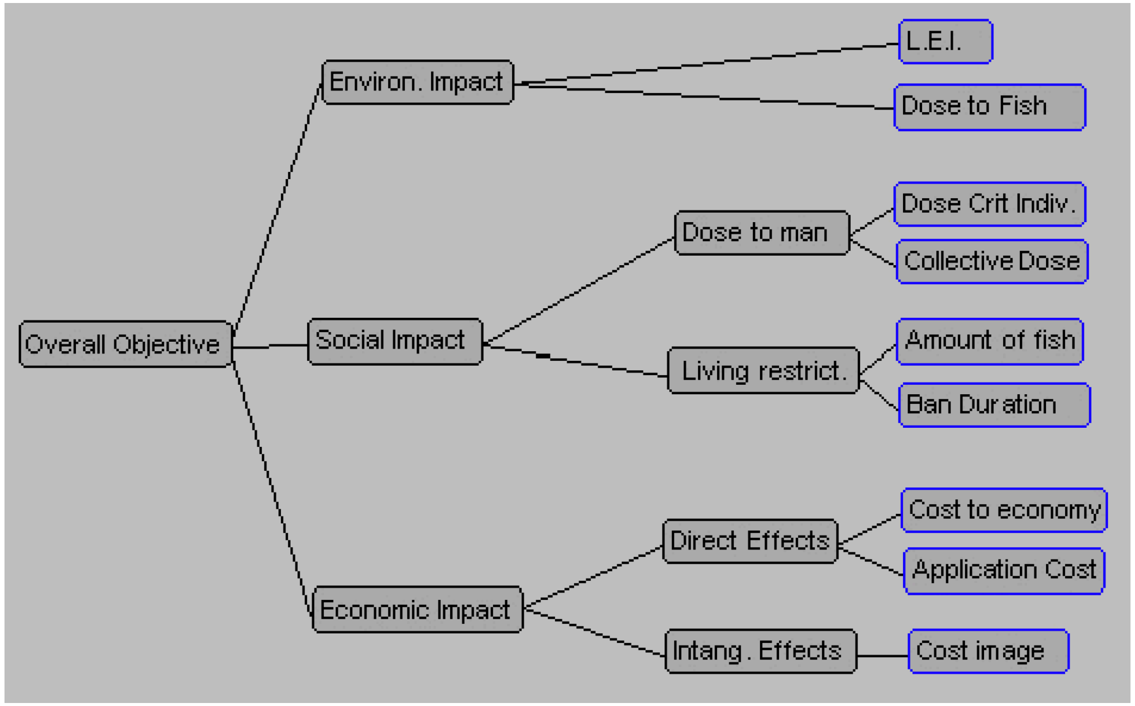

Table 1. Attributes with continuous scale

	Unit	Range
X_1: Lake Ecosystem Index	LEI	[1,5]
X_3: Dose Crit. Indiv.	mSv	[0,500]
X_4: Collective Dose	mSv $\times$ person	$[0,12 \times 10^4]$
X_5: Amount of Fish	Tonnes	[0,100]
X_6: Ban Duration	Months	[0,360]
X_7: Costs to Economy	Euros	$[0,10^8]$
X_8: Application Cost	Euros	$[0,10^7]$

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3. Countermeasures and their impacts

- A_1, A_2: *Potassium addition*
- A_3: *Lake liming*
- A_4, A_5: *Wetland liming*
- A_6: *Fertilization*
- A_7: *Removal of contaminated bottom sediments*
- A_8: *Treatment of contaminated fish and Bans on fish consumption*
- A_9: *No action*

Table 2. Countermeasure impacts

	A_1	A_2	A_3	A_4	A_5	A_6	A_7	A_8	A_9
X_1: L. Ecosyst. Index	1.33	1.33	1.332	1.630	1.618	1.962	1.33	1.33	1.33
X_3: Dose Crit. Indiv.	43.7	41.9	45.2	44.8	41.9	44.8	20.7	1.7	45.3
X_4: Collective Dose	3490	3360	3630	3590	3360	3560	1690	180	3630
X_5: Amount of Fish	0	0	0	0	0	0	0	36	0
X_6: Ban Duration	0	0	0	0	0	0	0	120	0
X_7: Costs to Economy	0	0	0	0	0	0	0	62300	0
X_8: Application Cost	2242	7830	1052	4209	11807	38	3156390	0	0

A 20% deviation was introduced in X_1 and X_8, a 10% for X_5 and X_7; and deviations ranging from -10% to +30% were used to derive the least and most impact for attributes X_3 and X_4, respectively.

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4. Quantifying DM's preferences: Experts

D. Cancio, former Head of the Public and Environmental Radiological Protection Unit (CIEMAT, *Centro de Investigaciones Energéticas, Medioambientales y Tecnológicas*

P. Carboneras, former Director of the Safety and Licensing Department at ENRESA (*Empresa Nacional de Residuos Radiactivos S.A.*)

E. Gallego, Professor of Nuclear Engineering at the UPM and public and environmental radiological protection expert.

Another expert assuming a more ecological role (DM_4) was also involved in the analysis.

4. Quantifying DM's preferences: attribute weights

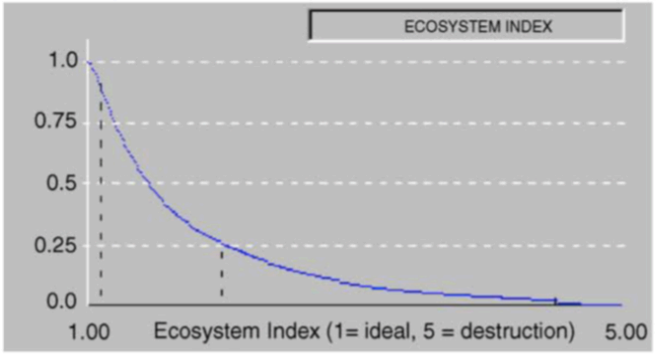
Table 5. Ordinal information concerning weights

	Ordinal information
DM_1	$w_3^1 > w_8^1 > w_9^1 > w_4^1 > \{w_5^1, w_6^1\} > w_7^1 > w_2^1 > w_1^1$
DM_2	$w_3^2 > w_4^2 > w_9^2 > w_6^2 > w_8^2 > w_7^2 > w_1^2 > w_2^2 > w_5^2$
DM_3	$\{w_3^3, w_4^3\} > w_8^3 > w_6^3 > w_9^3 > w_7^3 > \{w_1^3, w_2^3, w_5^3\}$
DM_4	$\{w_1^4, w_2^4\} > \{w_3^4, w_4^4\} > \{w_5^4, w_6^4\} > \{w_7^4, w_8^4\} > w_9^4$

Table 6. Ordinal information about the difference between the weights

DM	Ordinal information
DM_1	$\Delta_{1,\{3,8\}} > \Delta_{1,\{9,4\}} > \{\Delta_{1,\{6,7\}}, \Delta_{1,\{7,2\}}\} > \{\Delta_{1,\{8,9\}}, \Delta_{1,\{4,5\}}, \Delta_{1,\{2,1\}}\}$
DM_2	$\Delta_{2,\{1,2\}} > \Delta_{2,\{2,5\}} > \Delta_{2,\{4,9\}} > \{\Delta_{2,\{3,4\}}, \Delta_{2,\{9,6\}}, \Delta_{2,\{6,8\}}, \Delta_{2,\{8,7\}}, \Delta_{2,\{7,1\}}\}$
DM_3	$\Delta_{3,\{4,8\}} > \Delta_{3,\{8,6\}} > \Delta_{3,\{6,9\}} > \Delta_{3,\{9,7\}} > \Delta_{3,\{7,1\}}$

4. Quantifying DM's preferences: component utilities



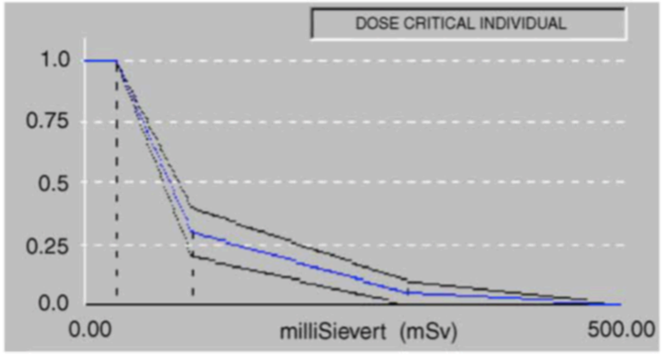


Table 3. Ordinal information concerning performances

		Ordinal information
X_2: Dose to Fish		$A_7 > \{A_2, A_3, A_4, A_5, A_6, A_8, A_9\} > A_1$
X_9: Cost to Image	DM_1	$\{A_1, A_2, A_3, A_4, A_5, A_6\} > A_8 > A_7$
	DM_2	$A_7 > \{A_1, A_2, A_3, A_4, A_5, A_6\} > A_8 > A_9$
	DM_3	$A_7 > \{A_2, A_5\} > \{A_1, A_3, A_4, A_6\} > A_8 > A_9$
	DM_4	$\{A_6, A_7, A_8\} > \{A_3, A_4\} > \{A_1, A_2, A_5\} > A_9$

Table 4. Ordinal information about the difference between the values

		Ordinal information
X_2: Dose to Fish		$\Delta_{2,\{7,2\}} > \Delta_{2,\{9,1\}}$
X_9: Cost to Image	DM_1	$\Delta_{9,\{8,7\}} > \Delta_{9,\{6,8\}}$
	DM_2	$\Delta_{9,\{7,1\}} > \Delta_{9,\{8,9\}} > \Delta_{9,\{6,8\}}$
	DM_3	$\Delta_{9,\{7,2\}} > \Delta_{9,\{8,9\}} > \{\Delta_{9,\{5,1\}}, \Delta_{9,\{6,8\}}\}$

4. Quantifying DM's preferences: veto values

Veto range: $[v_j^L, v_j^U]$ $\begin{cases} v_j^L = r_j^L, \text{ being } [r_j^L, r_j^U] \text{ the attribute range} \\ v_j^U = max_{s=1,\ldots,r}\{v_j^s\} \end{cases}$

Adjust range: $(a_j^L, a_j^U]$ $\begin{cases} a_j^L = v_j^U = max_{s=1,\ldots,r}\{v_j^s\} \\ a_j^U = max_{s=1,\ldots,k}\{v_j^s\} \end{cases}$

Veto function

$$v(A_i) = \prod_{j=1}^{n} v_j(A_i),$$

with $v_j(A_i) = \begin{cases} 1, & \text{if } x_{ij} > v_j^U \\ 0, & \text{if } x_{ij} \leq v_j^U \end{cases}$.

Adjust function

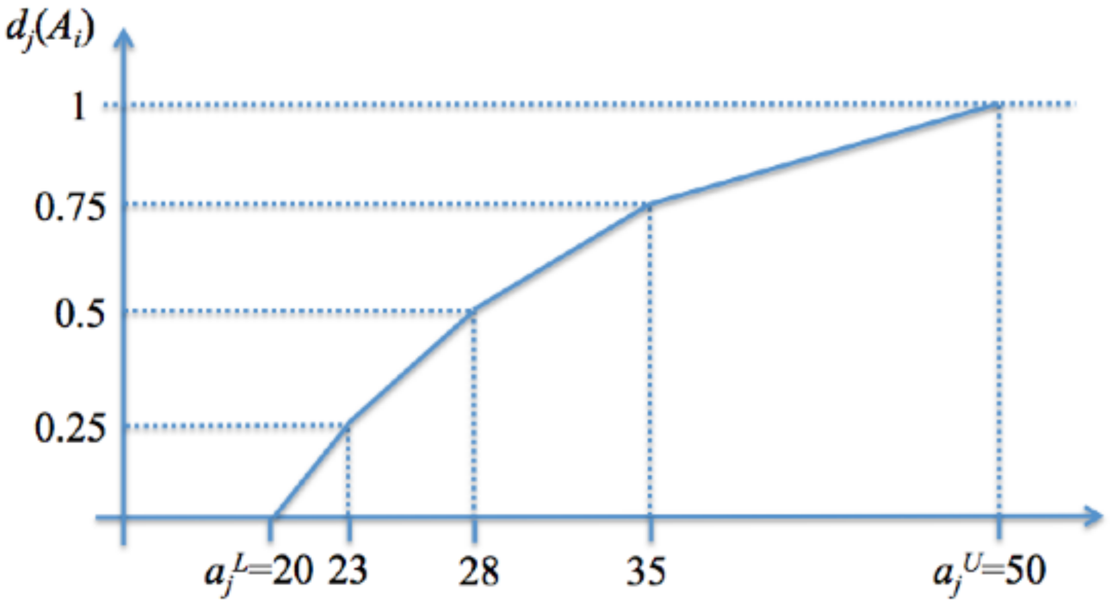
$$d_j(A_i) = \begin{cases} 1 & \text{if } x_{ij} > a_j^U \\ \frac{x_{ij} - a_j^L}{a_j^U - a_j^L} & \text{if } a_j^L < x_{ij} \leq a_j^U \\ 0 & \text{if } x_{ij} \leq a_j^L \end{cases}$$

4. Quantifying DM's preferences: veto values

Example (adjust function)

The adjust range is [20, 50], 50 being the highest veto value provided by the DMs.

Three of the $k - r$ less important DMs have provided the veto values 23, 28 and 35



4. Quantifying DM's preferences: veto values

Table 7. Veto values for DMs

DM	X_1	X_3	X_6
DM_1	-	100 mSv	24 months
DM_2	1.7 LEI	100 mSv	-
DM_3	2 LEI	-	-
DM_4	1.6 LEI	90 mSv	

Adjust functions

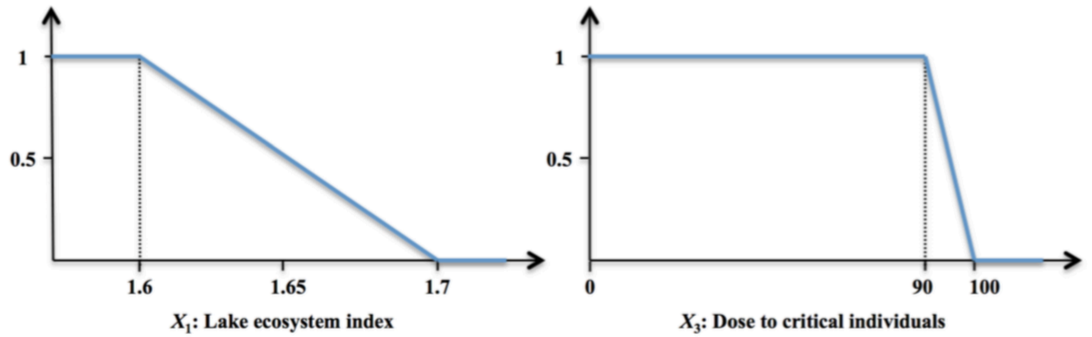


Fig. 4. Adjust functions for X_1 and X_3

Veto ranges

X_1: $[1.7, 5]$
X_3: $[100, 500]$
X_6: $[24, 360]$

A_6: *Fertilization*: its impact interval for X_1 is $[1.766, 2.354]$

A_8: *Treatment of contaminated fish and Bans on fish consumption*: its impact on X_6 is 36

5. Evaluation of alternatives: Extension of the Additive model

The adaptation of the additive multi-attribute utility function to account for the veto and adjust functions is:

$$u^s(A_i) = [\sum_{j=1}^{n} u_j(x_{ij}) w_j^s d_j(A_i)] \times v(A_i)$$

s refers to the s-th DM,

w_j^s is the weight of the j-th attribute for the s-th DM,

$v(A_i)$ is the value output by the veto function for the alternative A_i,

$d_j(A_i)$ is the value output by the adjust function for the alternative A_i in the attribute X_j, and

$u_j(x_{ij})$ is the component utility corresponding to the performance x_{ij}.

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5. Evaluation of alternatives: Dominance measuring methods

$$D^l = \begin{pmatrix} - & D^l_{12} & \cdots & D^l_{1(m-1)} & D^l_{1m} \\ D^l_{21} & - & \cdots & D^l_{2(m-1)} & D^l_{2m} \\ D^l_{31} & D^l_{32} & \cdots & D^l_{3(m-1)} & D^l_{3m} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ D^l_{m1} & D^l_{m2} & \cdots & D^l_{m(m-1)} & - \end{pmatrix} \quad \text{where} \quad \left[\begin{array}{l} D^l_{ks} = \quad min\{u^l(A_k) - u^l(A_s)\} \\ s.t. \\ \qquad u^l_j(x_{kj}), u^l_j(x_{sj}) \in U^l_j, j = 1, ..., n, \\ \qquad \mathbf{w}^l = (w^l_1, ..., w^l_n) \in W^l, \\ \text{with} \\ u^l(A_i) = [\sum_{j=1}^{n} u^l_j(x_{ij}) w^l_j d_j(A_i)] \times v(A_i). \end{array} \right.$$

$$D^1 = \begin{pmatrix} - & -0.1569 & -0.1746 & -0.1362 & -0.1042 & -0.1466 & -0.1272 \\ -0.1956 & - & -0.2363 & -0.2169 & -0.1866 & -0.2189 & -0.2607 \\ -0.1924 & -0.1806 & - & -0.2257 & -0.1179 & -0.0966 & -0.1823 \\ -0.2105 & -0.2034 & -0.1794 & - & -0.1949 & -0.1832 & -0.2814 \\ -0.2203 & -0.1662 & -0.1669 & -0.1895 & - & -0.1793 & -0.1953 \\ -0.2000 & -0.1484 & -0.1539 & -0.1570 & -0.0876 & - & -0.1591 \\ -0.1762 & -0.0972 & -0.1589 & -0.1914 & -0.1214 & -0.1139 & - \end{pmatrix}$$

$$D^3 = \begin{pmatrix} - & -0.1174 & -0.1475 & -0.2228 & -0.06158 & -0.1918 & -0.1567 \\ -0.3291 & - & -0.2416 & -0.1875 & -0.2513 & -0.2255 & -0.3403 \\ -0.2314 & -0.0780 & - & -0.1416 & -0.1506 & -0.1905 & -0.1855 \\ -0.1979 & -0.2553 & -0.2857 & - & -0.1075 & -0.3162 & -0.2131 \\ -0.2425 & -0.2714 & -0.2851 & -0.3056 & - & -0.2456 & -0.2689 \\ -0.2255 & -0.1777 & -0.2300 & -0.1620 & -0.1776 & - & -0.2113 \\ -0.1836 & -0.1836 & -0.2005 & -0.1743 & -0.1071 & -0.1094 & - \end{pmatrix}$$

$$D^2 = \begin{pmatrix} - & -0.1572 & -0.1553 & -0.2035 & -0.0298 & -0.1567 & -0.1713 \\ -0.3054 & - & -0.2034 & -0.2985 & -0.2120 & -0.3098 & -0.3407 \\ -0.2348 & -0.1991 & - & -0.1825 & -0.2397 & -0.3225 & -0.2338 \\ -0.2648 & -0.3035 & -0.2591 & - & -0.2763 & -0.3061 & -0.2371 \\ -0.3146 & -0.1970 & -0.3190 & -0.1922 & - & -0.2523 & -0.1846 \\ -0.1705 & -0.1095 & -0.2250 & -0.1849 & -0.1967 & - & -0.2134 \\ -0.2107 & -0.3127 & -0.2365 & -0.2039 & -0.1255 & -0.1981 & - \end{pmatrix}$$

$$D^4 = \begin{pmatrix} - & 0.0897 & -0.0707 & -0.0923 & 0.0078 & -0.0006 & -0.0099 \\ -0.1349 & - & -0.0302 & -0.0995 & -0.1138 & -0.0257 & -0.1489 \\ -0.1206 & -0.0193 & - & -0.0456 & 0.0249 & -0.0461 & -0.1712 \\ -0.0770 & -0.2456 & -0.1263 & - & -0.1095 & -0.2414 & -0.0706 \\ -0.0511 & -0.1594 & -0.1666 & -0.0631 & - & -0.0197 & -0.1137 \\ -0.0671 & -0.2201 & -0.1444 & 0.0183 & -0.0958 & - & -0.0525 \\ -0.1946 & -0.0665 & -0.1154 & -0.1351 & -0.1117 & -0.0043 & - \end{pmatrix}$$

5. Evaluation of alternatives: Dominance measuring methods

The *DMM* that we use derives a *global dominance intensity index* to rank alternatives on the basis that

$$D^l_{ks} \leq u^l(A_k) - u^l(A_s) \leq |D^l_{sk}|.$$

Then, we perform the following algorithm:

1. If $D^l_{ks} \geq 0$, then alternative A_k dominates A_s, and the dominance intensity of A_k over A_s is $DI^l_{ks} = d([D^l_{ks}, -D^l_{sk}], 0)$.
   Else ($D^l_{ks} < 0$):
   - If $D^l_{sk} \geq 0$, then A_s dominates A_k, and $DI^l_{ks} = -d([D^l_{ks}, -D^l_{sk}], 0)$.
   - Else ($D^l_{sk} < 0$),

$$DI^l_{ks} = [\frac{-D^l_{sk}}{-D^l_{sk} - D^l_{ks}} - \frac{-D^l_{ks}}{-D^l_{sk} - D^l_{ks}}] \times d([D^l_{ks}, -D^l_{sk}], 0).$$

2. Calculate a global dominance intensity (GDI^l) for each alternative A_k, i.e., $GDI^l_k = \sum_{s=1,\ s\neq k}^{m} DI^l_{ks}$, and rank the alternatives according to them.

The method incorporates the distance from the intervals $[D^l_{ks}, -D^l_{sk}]$ to 0 to account for their sizes and how far they are from 0.

5. Evaluation of alternatives: Dominance measuring methods

Table 8. Global dominance intensities and countermeasure rankings for DMs.

	DM_1	DM_2	DM_3	DM_4
1^{st}	$A_9(0.1162)$	$A_1(0.2398)$	$A_1(0.1904)$	$A_1(0.2932)$
2^{nd}	$A_1(0.1094)$	$A_7(0.1506)$	$A_3(0.1453)$	$A_3(0.1427)$
3^{rd}	$A_3(0.0235)$	$A_9(0.0278)$	$A_9(0.1419)$	$A_2(0.0163)$
4^{th}	$A_7(0.0125)$	$A_3(-0.0051)$	$A_7(0.0305)$	$A_9(-0.0494)$
5^{th}	$A_4(-0.0419)$	$A_4(-0.1158)$	$A_4(-0.0492)$	$A_7(-0.0843)$
6^{th}	$A_5(-0.1006)$	$A_2(-0.1362)$	$A_2(-0.1811)$	$A_5(-0.1022)$
7^{th}	$A_2(-0.1193)$	$A_5(-0.1611)$	$A_5(-0.2778)$	$A_4(-0.2164)$

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6. Aggregating the rankings to derive a group ranking

Stochastic optimization search

Kemeny optimal aggregation (Kemeny, 1959): optimizes the average Kendall distances between a candidate aggregate list and each of the input lists.

Computing the Kemeny optimal aggregate is **NP-hard**

Stochastic search algorithms based on the *cross entropy Monte Carlo approach*

Order explicit algorithm (Lin and Ding, 2009)

Provides an alternative for finding an optimal solution while circumventing the combinatorial nature of the problem.

6. Aggregating the rankings to derive a group ranking

Table 8. Global dominance intensities and countermeasure rankings for DMs.

	DM_1	DM_2	DM_3	DM_4	Group
1^{st}	$A_9(0.1162)$	$A_1(0.2398)$	$A_1(0.1904)$	$A_1(0.2932)$	A_1
2^{nd}	$A_1(0.1094)$	$A_7(0.1506)$	$A_3(0.1453)$	$A_3(0.1427)$	A_9
3^{rd}	$A_3(0.0235)$	$A_9(0.0278)$	$A_9(0.1419)$	$A_2(0.0163)$	A_3
4^{th}	$A_7(0.0125)$	$A_3(-0.0051)$	$A_7(0.0305)$	$A_9(-0.0494)$	A_7
5^{th}	$A_4(-0.0419)$	$A_4(-0.1158)$	$A_4(-0.0492)$	$A_7(-0.0843)$	A_4
6^{th}	$A_5(-0.1006)$	$A_2(-0.1362)$	$A_2(-0.1811)$	$A_5(-0.1022)$	A_2
7^{th}	$A_2(-0.1193)$	$A_5(-0.1611)$	$A_5(-0.2778)$	$A_4(-0.2164)$	A_5

- A_1, A_2: *Potassium addition*
- A_3: *Lake liming*
- A_4, A_5: *Wetland liming*
- A_6: *Fertilization*
- A_7: *Removal of contaminated bottom sediments*
- A_8: *Treatment of contaminated fish and Bans on fish consumption*
- A_9: *No action*